Solutions to Workbook-2 [Mathematics] | Permutation & Combination

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27.(C) There are two possible cases

Case I Five 1's one 2's, one 3's

Number = $\frac{7!}{5!}$ = 42

Case II Four 1's, three 2's

Number = $\frac{7!}{4!\,3!} = 35$ ∴ Total number = 42 + 35 = 77

28.(A) Since, a five-digit number is formed using the digits {0, 1, 2, 3, 4 and 5} divisible by 3 i.e. only possible when sum of the digits is multiple of three.

Case I Using digits 0, 1, 2, 3, 4, 5

Number of ways = 4 × 4 × 3 × 2 × 1 = 96

Case II Using digits 1, 2, 3, 4, 5

Number of ways = 5 × 4 × 3 × 2 × 1 = 120

∴ Total number formed = 120 + 96 = 216

29. $^9P_4 \times {}^9P_3(11)!$

Let the two sides be *A* and *B*. Assume that four particular guests wish to sit on side *A*. Four guests who wish to sit on side *A* can be accommodated on nine chairs in $^9P_4$ ways and three guests who wish to sit on side *B* can be accommodated in $^9P_3$ ways. Now, the remaining guests are left who can sit on 11 chairs on both the sides of the table in (11!) ways. Hence, the total number of ways in which 18 persons can be seated = $^9P_4 \times {}^9P_3 \times (11)!$.

30.(C) Since, r, s, t are prime numbers.

∴ Selection of p and q are as under

p	q	Number of ways
r^0	r^2	1 way
r^1	r^2	1 way
r^2	r^0, r^1, r^2	3 ways

∴ Total number of ways to select, $r = 5$

Selection of s as under

s^0	s^4	1 way
s^1	s^4	1 way
s^2	s^4	1 way
s^3	s^4	1 way
s^4		5 ways

∴ Total number of ways to select s = 9

Similarly, the number of ways to select $t = 5$

∴ Total number of ways = 5 × 9 × 5 = 225

31.(n = 5)

Number of line segment joining pair of adjacent point = n

Number of line segment obtained joining n points on a circle = $^nC_2$

Number of red line segments = ${}^nC_2 - n$

Number of blue line segments = n $\therefore$ ${}^nC_2 - n = n \Rightarrow \dfrac{n(n-1)}{2} = 2n \Rightarrow n = 5$

32. $n^n, n!$ Let $A = \{x_1, x_2, \ldots\ldots, x_n\}$

$\therefore$ Number of function from A to A is n^n and out of these

$\sum_{r=1}^{n} (-1)^{n-r}\ {}^nC_r (r)^n$ are onto functions.

33. $2^n - 1$ The number of students answering exactly $k(1 \le k \le n-1)$ questions wrongly is $2^{n-k} - 2^{n-k-1}$. The number of students answering all questions wrongly is 2^0.

Thus, total number of wrong answers $= 1(2^{n-1} - 2^{n-2}) + 2(2^{n-2} - 2^{n-3}) + \ldots + (n-1)(2^1 - 2^0) + 2^0 \cdot n$

$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2^1 + 2^0 = 2^n - 1$

34. **(i) 2702 (ii) 1008**

Given that, there are 5 women and 8 men, a committee of 12 is to be formed including at least 5 women.

This can be done in = (5 women and 7 men) + (6 women and 6 men) + (7 women and 5 men) + (8 women and 4 men)

Total number of ways of forming committee

$= ({}^9C_5 \cdot {}^8C_7) + ({}^9C_6 \cdot {}^8C_6) + ({}^9C_7 \cdot {}^8C_5) + ({}^9C_8 \cdot {}^8C_4) + ({}^9C_9 \cdot {}^8C_3)$

$= 1008 + 2352 + 2016 + 630 + 56 = 6062$

(i) The women are in majority = 2016 + 630 + 56 = 2702 (ii) The men are in majority = 1008 ways

35.(*n*=3) Since, student is allowed to select at most n books out of $(2n+1)$ books.

$\therefore$ ${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 63$ ...(i)

We know ${}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_{2n+1} = 2^{2n+1}$

$\Rightarrow$ $2({}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n) = 2^{2n+1}$

$\Rightarrow$ ${}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = (2^{2n} - 1)$ ...(ii)

From Eqs. (i) and (ii), we get $2^{2n} - 1 = 63$ $\Rightarrow$ $2^{2n} = 64$ $\Rightarrow$ $2n = 6 \Rightarrow n = 3$

36.(64) **Case I:** When one black and two other balls are drawn. $\Rightarrow$ Number of ways = ${}^3C_1 \cdot {}^6C_2 = 45$

Case II: When two black and one other balls are drawn $\Rightarrow$ Number of ways = ${}^3C_2 \cdot {}^6C_1 = 18$

Case III: When all three black balls are drawn $\Rightarrow$ Number of ways = ${}^3C_3 = 1$

$\therefore$ Total number of ways = 45 + 18 + 1

37.(485) The possible cases are

Case I: A man invites 3 ladies and woman invites 3 gentlemen.

Number of ways = ${}^4C_3 \cdot {}^4C_3 = 16$

Case II: A man invites (2 ladies) and woman invites (2 gentlemen).

Number of ways = $({}^4C_2 \cdot {}^3C_1, {}^3C_1 \cdot {}^4C_2) = 324$

Case III: A man invites (1 lady, 2 gentlemen) and woman invites (2 ladies, 1 gentleman).

Number of ways = $({}^4C_1 \cdot {}^3C_2) \cdot ({}^3C_2 \cdot {}^4C_1) = 144$

Case IV: A man invites (3 gentlemen) and woman invites (3 ladies).

Number of ways = ${}^3C_3 \cdot {}^3C_3 = 1$ $\therefore$ Total number of ways = 16 + 324 + 144 + 1 = 485

38. Since, m men and n women are to be seated in a row so that no two women sit together. This could be shown as $\times M_1 \times M_2 \times M_3 \times \ldots \times M_m \times$

Which shows there are $(m+1)$ places for n women.

$\therefore$ Number of ways in which they can be arranged = $(m)!\,{}^{m+1}P_n = \dfrac{(m)!\cdot(m+1)!}{(m+1-n)!}$